WHITE PAPER

Toshiba Teli Corporation
4-7-1, Asahigaoka, Hino, Tokyo 191-0065, Japan

You should know
About Magnification of Taking Lens

In the machine vision field, optical magnification is important to select taking lens or to consider optical system.
However, it is not well known and sometimes confused that magnification has a meaning other than imaging magnification, generally known, which indicates size ratio of object and image.
We would like to explain, here, a concept and applications of magnification clearly.

■ Optical Magnification

Optical Magnification with Taking Lens

Ratio of length change by optical system is called "magnification".
Taking lenses used in photo or machine vision are included in "Image forming optical system".
In the term of "magnification m", there are "lateral Magnification β" and "longitudinal magnification α". β indicates ratio in size of concerned object and image while α indicates ratio in optical axis. These will be explained in next chapter.
In case of magnification in size, it is called "equal magnification", "1-fold magnification" or "1x" if image and object are the same in size. The value is more than 1 such as "2-fold magnification", "2 power" or "2x" in case of expansion while it is less than 1 such as "0.5-fold magnification" or "0.5x" in case of shrinking.
Furthermore, the sign of magnification (Lateral magnification, β) is negative (-: minus) as image is usually inverted.

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Magnification of Telescope and Binoculars

As taking lens in image forming optical system shoots real image, "Lateral magnification β", size ratio, can be calculated by direct measurement of object and image. On the other hand, in case of tele scope and binoculars, real image made by objective lens is seen as virtual image through eyepiece. They are in a special optical system as incident light and outgoing light is parallel to optical axis. In this optical system, called "afocal system" or "no focus optical system", "angular magnification γ", which is a ratio of inclination angle u between light from object and optical axis to inclination angle u' between light to image and optical axis, is used for magnification m.[1]

$$\gamma = \frac{\tan u'}{\tan u}$$

This means that object can be seen in the same size as if observer comes close to it in 1/γ distance.

As magnification gets bigger, the structure becomes bigger as well as viewable range becomes narrower and image becomes darker because "exit pupil" becomes smaller.

Binoculars with specifications of 6 to 8-fold magnification and 30 – 42 mm in diameter[2] is manly used for bird watching for its portability. While that with specifications of 7 – 18-fold magnification and 50 – 70 mm in diameter is mainly used for star watching. Binoculars of high magnification are usually used with tripod to mount. Meanwhile, that of around 10-fold magnification with stabilizing mechanism is getting popular for theater use recently.

Exit pupil is calculated with formula as "lens diameter ÷ magnification", and it get hard to see if it is smaller than eye pupil. Because of this reason, binoculars with exit pupil of more than 3 mm are used in daytime, and that with more than 5 mm is used in nighttime or dark place. Binoculars with compact size and high magnification are sold as popular type. Due to smaller exit pupil, their view might be dark or visibility might be lost by eye movement (black out). Careful use is required for these types of binoculars.

Magnification of Magnifying Glass

Magnifying glass is also in optical system to see virtual image. Magnification m can be calculated as a ratio of "distance of distinct vision of 250 mm" and "focal length of f".

$$m = \frac{250}{f}$$

[1] No explanation chart here due to limited paper space. It is recommended to refer the tele scope antakingd binoculars related

Web for your better understanding.

[2] In case of binoculars with 8 power magnification and 30 mm diameter, it is described as "8 X 30" and called "eight by thirty".

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"Distance of distinct vision" is the distance between eyes and object in which we can see the object clearly without tension or fatigue. It is supposed to be 250 mm (=4 diopter) since ancient age. The reason that 25 – 30 cm distance is recommended for book reading is also distinct vision distance. By the way, elasticity and adjustability of eye lens is getting lower as aging and object can hardly be seen unless locating it further than distance of distinct vision. This is so called Presbyopia.

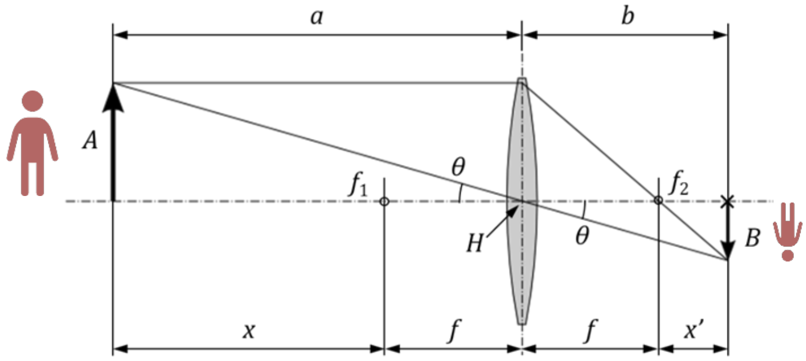
Magnification of Zoom Lens

In case of zoom lens, used in digital camera or video camera, magnification of 3x, 6x or 30x are indicated. This is called "zoom ratio" and it means ratio between focal length of longer side (tele side) and shorter side (wide side). For example, zoom ratio of zoom lens which focal length can be adjusted 8 – 80 mm is 10x.

■ Lateral Magnification and Longitudinal Magnification

Lateral Magnification

In general, optical magnification means "lateral magnification β".



Lateral magnification β is the ratio of image height B against object height A.

$$\beta = \frac{B}{A}$$

The relation of object distance x and delivery amount is described by following formula.

$$x = \frac{f}{\beta}$$

$$x' = f \cdot \beta$$

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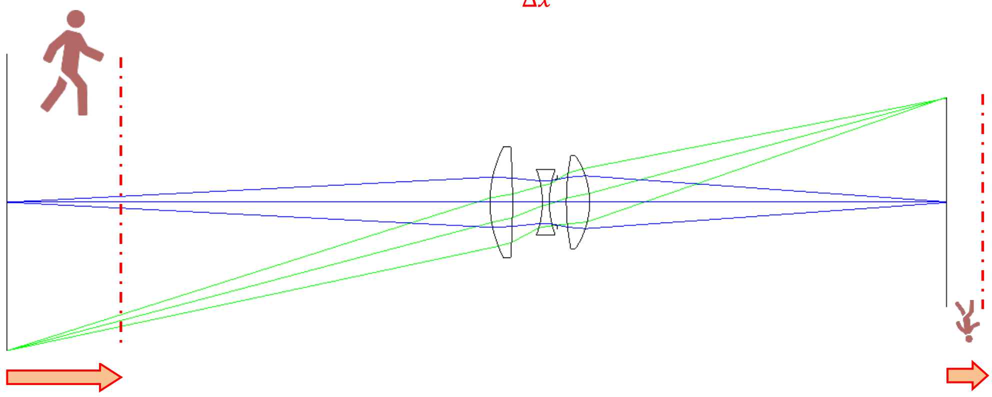
Longitudinal Magnification

"Longitudinal magnification α" is a ratio of "minute transfer amount ⊿x'" of image against "minute transfer amount ⊿x" of object along optical axis. In other words, it means sensitivity on focus out.

$$\alpha = \frac{\Delta x'}{\Delta x}$$



"Longitudinal magnification α" can be calculated by Differentiating Newton formula.

$$x \cdot x' = -f^2$$

$$x' = \frac{-f^2}{x} = -f^2 \cdot x^{-1}$$

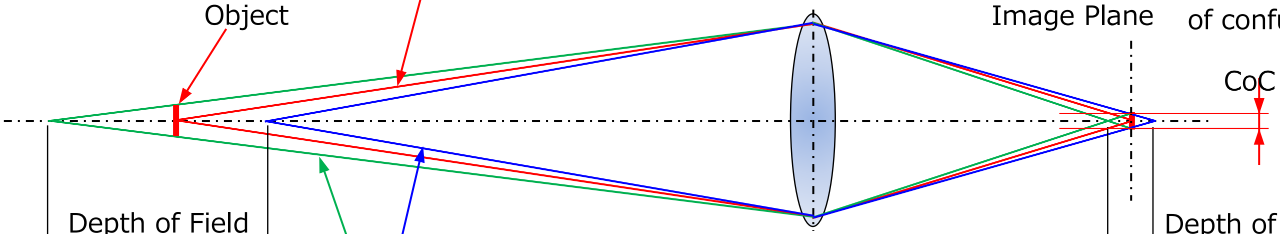
$$\frac{\Delta x'}{\Delta x} = -f^2(-x^{-2}) = \frac{f^2}{x^2} = \beta^2 = \alpha$$

Therefore, longitudinal magnification α is the square of lateral magnification β in case amount of object movement is comparatively small.

■ Depth of Focus and Depth of Field

Formula of Depth of Field

In case of general object distance in machine vision, the formula can be easily found with above mentioned "longitudinal magnification α" because "depth of field DoF_i" is "depth of focus DoF_o" which is shifted to object side through lens.



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”Depth of Focus DoF_o” can be found with “permissible circle of confusion CoC” and “effective F value F_{eff}”.

$$DoF_o = 2 \cdot CoC \cdot F_{eff}$$

”Effective F value F_{eff}” can be found with following formula depending on the object distance of finite or infinite.

in case of infinite distance： $F_{eff} = F_{no} = \frac{f}{D}$

in case of finite distance ： $F_{eff} = F_{no}(1+\beta)$

Let “permissible circle of confusion CoC” be larger one of “pixel pitch P_{pix}” or “airy disk diameter D_{airy}”.

”Depth of field DoF_i” can be found with dividing “depth of focus DoF_o” by “longitudinal magnification α”.

$$DoF_i \approx \frac{DoF_o}{\alpha} = \frac{DoF_o}{\beta^2} = \frac{2 \cdot CoC \cdot F_{eff}}{\beta^2}$$

According to this formula, the smaller the F number (brighter) of lens, the smaller the pixel pitch of sensor and the larger the optical magnification (enlargement) the narrower the depth of field.

Incidentally, “depth of field DoF_i” is applicable to short object distance case such as machine vision, however, the depth is significantly different between the front side and the back side of the object in long object distance case such as surveillance camera.
In this case, “depth of field DoF_i” is found with “depth of focus DoF_o” and “Newton’s lens formula”[3].

[3] Object distance x can be found by assuming width in one side of “depth of focus DoF_o” (DoF_o=2・CoC・F_{eff}) to be delivery amount x’. No formula is shown here due to paper space.